LEVEL II

AD _____

RDTE Project No. _____

TECOM Project No. _____

DPG Document No. DPG-TR-81-001

Test Sponsor _____

Test Sponsor No. _____

TRADOC AC No. _____

AN ANALYTIC SOLUTION

FOR SURFACE SOURCE

SIGMA Z CALCULATIONS

TECHNICAL REPORT

BY

CHRISTOPHER BILTOFT

JANUARY 1981

**U.S. ARMY DUGWAY PROVING GROUND**

**Dugway, Utah 84022**

APPROVED FOR PUBLIC RELEASE:

DISTRIBUTION UNLIMITED

81 3 06 040

| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|---|---|---|
| 1. REPORT NUMBER | 2. GOVT ACCESSION NO. AD-A096 04Z | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE *(and Subtitle)* AN ANALYTIC SOLUTION FOR SURFACE SOURCE SIGMA Z CALCULATIONS | | 5. TYPE OF REPORT & PERIOD COVERED Technical Report |
| | | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s) Christopher A. Biltoft | | 8. CONTRACT OR GRANT NUMBER(s) |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS U. S. Army Dugway Proving Ground ATTN: STEDP-MT-DA-M Dugway, UT 84022 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 3052900111 |
| 11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Dugway Proving Ground Dugway, UT 84022 | | 12. REPORT DATE January 1981 |
| | | 13. NUMBER OF PAGES |
| 14. MONITORING AGENCY NAME & ADDRESS*(If different from Controlling Office)* | | 15. SECURITY CLASS. *(of this report)* Unclassified |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Richardson number, bulk Richardson number, sigma z, crosswind integrated dosage, Project Prairie Grass, BC 412, Gaussian diffusion equations, sigma w, vertical turbulence, sigma e, correction factor, Pasquill categories, fractional error, dimensionless height, roughness length, friction velocity, NRTS, Kansas 1968 Field Program, arctangent equation, transport wind speed, meteorological profile

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

An analytic method for calculating the standard deviation of cloud material vertical distribution (sigma z) has been developed. This method treats sigma z for a surface source as a function of travel distance and the vertical component of atmospheric turbulence, modified by a correction factor which represents the degree of sigma z departure from rectilinear expansion. The vertical component of atmospheric turbulence during lapse conditions is given as a function of stability parameters obtained from measured meteorological profile data. A regression equation was also developed to present

DD $_{1 \text{ JAN } 73}^{\text{FORM}}$ 1473 EDITION OF 1 NOV 65 IS OBSOLETE

19. continued.
cloud dimensions, dimensionless wind shear, continuity, deposition, diabatic
influence function, dimensionless temperature gradient

20.  continued.

cont

turbulence as a function of stability for inversion conditions.  The correction
factor assumes the form of an arctangent equation which expands over a
continuum of stability and travel distance.  The correction factor equation
fitted to Project Prairie Grass data has been successfully employed on
independent sets of diffusion data collected at Dugway Proving Ground, Utah,
and the National Reactor Testing Station, Idaho.

## TABLE OF CONTENTS

## SUMMARY

An analytic method for calculating the standard deviation of cloud material vertical distribution (sigma z) for a surface source over short to moderate travel distances has been developed. The surface source sigma z is described as the product of travel distance and the vertical component of atmospheric turbulence, modified by a correction factor (i). The resultant sigma z equation permits calculation of cloud vertical growth, from meteorological profile data, over a continuum of stability and travel distance.

The correction factor, which is a measure of departure from rectilinear cloud growth, assumes the form of an arctangent equation. The general form of the arctangent equation is $i = (-ATAN(a \cdot BU + b) + c)/d$. Stability is represented by bulk Richardson number (BU), and coefficient a is a constant. Variables b, c, and d are functions of travel distance. Coefficients a, b, c, and d were chosen to provide a best fit field of i values which give sigmas satisfying Gaussian equations for Project Prairie Grass time averaged crosswind integrated concentration data.

The vertical component of atmospheric turbulence for lapse conditions is shown to be a function of a stability parameter, z/L, which is obtained from meteorological profile data. During inversion conditions, turbulence is described by means of a regression equation using bulk Richardson number.

The sigma z computation procedure has been successfully used on independent sets of diffusion data collected at the U. S. Army Dugway Proving Ground, Utah, and the National Reactor Testing Station (NRTS), Idaho.

# I. INTRODUCTION

Gaussian cloud models used for the prediction of dosages or concentrations downwind from a surface source are critically dependent on the determination of a representative transport wind speed ($\tilde{u}$) and on an adequate estimation of cloud dimensions. Cloud dimensions in Gaussian models are represented by cloud material standard deviations (sigmas). Gaussian models are useful for prediction if the $\tilde{u}$ and sigmas can be estimated from meteorological profile data. Transport wind speed can be readily estimated from measured wind profile data if the cloud vertical dimensions are known, but present procedures for estimating cloud dimensions from meteorological profile data are not adequate. For a surface source, calculation of cloud material vertical standard deviation (sigma z) has been particularly difficult due to the steep vertical gradients of temperature, wind speed, and turbulence which frequently occur near the Earth's surface.

This report describes the development of an analytic method for using estimates obtained from meteorological profile data of the vertical component of atmospheric turbulence to calculate sigma z. Sigma z is the product of travel distance and the vertical turbulence estimate, modified by a correction factor (i). This correction factor accomodates departures from Gaussian assumptions and turbulence measurement deficiencies over a continuum of stability and downwind travel distance.

# II. DEFINITIONS

The following symbols for standard variables and dimensionless quantities are defined for use in the discussion to follow:

$g = 9.8$ m/sec$^2$      acceleration due to gravity

$k = 0.41$      von Karman's constant (dimensionless)

$z$, meters      height of measurement above the surface

$z_0$, meters      roughness length

$u, w$, m/sec      longitudinal and vertical wind components

$\rho$, g/m$^3$      air density

$T$, degrees Kelvin      absolute air temperature

$\theta$, degrees Kelvin      potential temperature

$u_*$, m/sec      friction velocity

$\theta_* = \overline{w\theta}/ku_*$      scaling temperature (degrees Kelvin)

$z/L = kg\overline{w\theta}z/\bar{\theta}u_*^3$      dimensionless height, a stability parameter

2

$Ri = (g/\bar{\theta})(\overline{\partial\theta/\partial z})/(\overline{\partial u/\partial z})^2$     Richardson number, dimensionless

$BU = 100z^2(g/\bar{\theta})(\overline{\partial\theta/\partial z})/u^2$     bulk Richardson number, dimensionless

$\phi_m = (kz/u_*)(\overline{\partial u/\partial z})$     dimensionless wind shear

$\phi_h = (z/\theta_*)(\overline{\partial\theta/\partial z})$     dimensionless temperature gradient

An overbar indicates a time averaged quantity and a prime denotes fluctuations about the average. The Ri and BU are stability parameters which apply at the geometric mean height between top and bottom of the layer considered.

## III. THE GAUSSIAN EQUATION, TRANSPORT WIND, AND SIGMA Z

The analytic method was developed by satisfying the Gaussian crosswind integrated dosage equation transport wind and sigma z requirements using meteorological profile data. Equation (1) is a Gaussian equation for the crosswind integrated dosage of non-depositing tracer material measured at height z. Total reflection from the surface is assumed using the method of images presented by Sutton (1953).

$$CWID = \frac{Q[EXP(-\frac{1}{2}((z-h)/\sigma_z)^2)+EXP(-\frac{1}{2}((z+h)/\sigma_z)^2)]}{(2\pi)^{\frac{1}{2}}\sigma_z\tilde{u}} \qquad (1)$$

where:

CWID = crosswind integrated dosage $(mg \cdot sec \cdot m^{-3})$

Q    = source strength (milligrams)

h    = release height (meters)

$\sigma_z$   = sigma z

Strong gradients of wind speed frequently exist near the ground. However, if the distribution of the wind speed-dosage product $(D_i u_i dz)$ is normal with height, then equation (2) defines a weighted mean wind speed which can be used as a transport wind without violating the assumptions needed to integrate the Gaussian equations. Estimates of transport wind

$$\tilde{u} = \int Dudz/\int Ddz \qquad (2)$$

speed from meteorological profile data were obtained using equation (3) developed by H. E. Cramer et al. (1972). Equation (3) uses the 2-meter wind speed for $u_r$ during daytime (lapse) conditions, and the 1-meter wind

speed for nighttime (surface based inversion) conditions. Variable $z_r$ is the height of the $u_r$ measurement and $2.15\sigma_z$ is the "top" of the cloud, or the height at which cloud dosage is reduced to one tenth of its centroid value. Exponent p is the wind speed profile exponent taken from $z_r$ to 16 meters.

$$\tilde{u} = \frac{u_r[(2.15\sigma_z)^{(1+p)} - z_r^{(1+p)}]}{(2.15\sigma_z - z_r)z_r^p(1+p)} \tag{3}$$

With $\tilde{u}$ specified by equation (3) and the other variables measured, equation (1) can be used to solve for sigma z by means of a Newton-Raphson iterative method. But before this is done, a careful examination of continuity considerations is required. Because diffusion problems involve the release of a gas or a large number of minute particles, the source strength is often imprecisely known. A further continuity consideration is the deposition rate or physical decay for materials which have limited residence time in the atmosphere. With a passive gas or minute particles traveling over short distances, it can be assumed that the total amount of material released into the atmosphere remains airborne. For other materials, diffusion equations must be modified to include decay or deposition terms. Sampler collection efficiencies and losses due to data handling and assay techniques are also continuity considerations. Unfortunately, these factors are often not carefully specified for diffusion experiments. Because of the difficulties in satisfying continuity considerations, the amount of high quality data providing information required for the solution of equation (1) is limited. Project Prairie Grass data (Haugen, 1959) collected in 1957 at O'Neill, Nebraska, were chosen for equation (1) sigma z solutions because of the quality and quantity of the data.

IV. PROJECT PRAIRIE GRASS

Project Prairie Grass was a field diffusion program designed to investigate diffusion from a continuous point source over flat terrain. For a 5 to 6 centimeter grass cover, Pasquill (1978) estimated a surface roughness of 0.8 centimeters. The source consisted of a nozzle emitting sulphur dioxide ($SO_2$) at a height of either 0.5 or 1.5 meters above ground level. Source duration was ten minutes for each diffusion trial. Samplers set out in arcs 50, 100, 200, 400, and 800 meters from the source provided SO2 concentration data measured at a height of 1.5 meters. The 100-meter arc also included a vertical sampler array for gathering cloud vertical concentration measurements.

The question of continuity for Project Prairie Grass data was treated through careful source strength measurements and by assuming that the diffusing $SO_2$ remained airborne as it moved across the sampling grid. Barad and Haugen (1959) and Elliott (1961) considered the question of deposition on vegetation and concluded that virtually all of the released $SO_2$ remained airborne during sampling. Additional continuity considerations included sampler efficiencies and assay techniques. Barad (1958a) presented

4

an extensive discussion of data collection and assay techniques used. He
concluded that, for the $SO_2$ sampler measurements, there was a cumulative
error of about ten percent. Sample budget calculations from the 100-meter
arc vertical array also indicate very efficient tracer recovery.

Time-averaged $SO_2$ concentration measurements taken at equally spaced
positions along each arc were used to compute CWID. The collected concen-
tration data were averaged over a ten-minute period for the data report.
Multiplying the reported sampler data by the ten-minute averaging period
produced a total dosage. The sum of total dosages across each arc multiplied
by sampler spacing intervals produced CWIDs which were then used in equation
(1) to obtain values of sigma z.

Unfortunately, cloud centroid measurements were not made at each arc
during Project Prairie Grass. Except for the 100-meter arc, no attempt was
made to measure vertical concentration profiles. Consequently, the behavior
of the cloud centroid as it moved beyond 100 meters is not known. To solve
equation (1) it was necessary to assume no cloud centroid rise beyond the
100-meter arc. Although necessary to obtain solutions, this assumption is
not well founded. Willis and Deardorff (1976), for example, have documented
rapidly decreasing surface concentrations in unstable conditions due to
thermally induced cloud centroid rise. It is probable that cloud centroid
rise affected Project Prairie Grass sampler measurements, especially during
unstable conditions at the more distant arcs.

Meteorological profile data accompanying Project Prairie Grass
diffusion trials were collected on 16-meter towers, each instrumented for
wind speed, temperature, and wind direction measurements. Also, bi-vanes
for measurement of horizontal and vertical wind angles were mounted on a
series of 2-meter masts. These data were collected using a one-second
averaging time for twenty minutes. Data processing from magnetic tapes
produced temperature gradients, velocity profiles, and wind angle standard
deviation information. The data and further details on Project Prairie Grass
are in Barad (1958a,b) and Haugen (1959).

V. CHOOSING A TURBULENCE PARAMETER

Turbulence data accompanying the diffusion trials at Project Prairie
Grass consist of bi-vane horizontal and vertical wind angle measurements,
presented as wind angle sigmas. However, the use of vertical wind angle
sigmas derived from these bi-vanes is not widely accepted. The primary
objection to these data is that the vanes were mounted on 2-meter masts.
Because of proximity to the ground, it would appear that some of the
turbulent energy would be truncated and the data would therefore not
represent the full amount of vertical turbulence acting on a diffusing cloud.
This truncation would be most noticeable on the larger eddies generated in
the unstable regime.

Given the limitations of the bi-vane sigmas and the fact that only 16
of the 34 unstable regime trials had tabulated sigmas, an alternative means
of estimating unstable regime vertical turbulence was chosen. This led to
an investigation of meteorological profile data collected during the Kansas

1968 Field Program (Izumi, 1971). This project was carried out in the summer of 1968 at a field site in southwest Kansas. The Kansas data include drag plate measurements for $u_*$ and research grade wind and temperature profile measurements on a 32-meter tower. Measurements of turbulent velocity fluctuations and air temperature fluctuations were made at three levels concurrently with profile measurements. The basic averaging period for all data was 15 minutes. Turbulence data derived from these measurements do not suffer from the deficiencies attributed to the 2-meter bi-vane data.

A relationship was established between the standard deviations of the vertical velocity ($\sigma_w$) and Kansas unstable regime meteorological profile data using the following assumptions:

1. A log-linear wind velocity profile (equation 4) describes the horizontal wind speed as a function of friction velocity, the height to roughness logarithm, and a diabatic influence function ($\psi$). The unstable

$$u = u_*(\ln(z/z_0) - \psi)/k \tag{4}$$

regime diabatic influence function as defined by Paulson (1970) is presented in equation (5).

$$\psi = 2\ln((1+\phi_m^{-1})/2) + \ln((1+\phi_m^{-2})/2) - 2\tan^{-1}\phi_m^{-1} + \pi/2 \tag{5}$$

2. Empirical equations (6) and (7) from Dyer (1974) estimate the unstable regime dimensionless wind shears and temperature gradients.

$$\phi_m = (1 - 16(z/L))^{-\frac{1}{4}} \tag{6}$$

$$\phi_h = (1 - 16(z/L))^{-\frac{1}{2}} \tag{7}$$

Several similar equations for these variables are found in the literature. Equations (6) and (7) were chosen in the given form because they not only fit the micro-meteorological data collected at Hay and Gurley, New South Wales, Australia (Dyer and Hicks, 1970), and provide a reasonable fit to the unstable regime Kansas data as corrected by Wieringa (1980).

Equations (6) and (7) imply equality between Ri and z/L in the unstable regime. Equations (8) and (9) thus describe the relationships between the stability parameters.

$$Ri = (z/L)(\phi_h/\phi_m^2) = z/L \tag{8}$$

$$BU = 100(z/L)/((1 - 16(z/L))^{\frac{1}{2}}(\ln(z/z_0) - \psi)) \qquad (9)$$

Equations (6) through (9) are valid for a z/L range of $0 > z/L > -1.0$. Beyond a z/L of -1.0, Ri becomes steadily larger than z/L. A sufficient amount of extremely unstable data is not available for further analysis of this phenomenon.

With the relationships given by equations (4) through (6), it was possible to establish a link between Kansas unstable regime profile data and sigma w. This was accomplished by examining the ratio of $\sigma_w$ to $u_*$. This ratio would approach unity except for the contribution of buoyancy ($g\overline{w\theta}/\bar{\theta}$) to vertical turbulent motions. Using Wieringa's (1980) suggested adjustments to the Kansas z/L data, it was found that the ratio of $\sigma_w$ to $u_*$ is approximated by the inverse of dimensionless wind shear. Equation (10) also provides a close fit to the extremely unstable data attributed to Monji (Businger, 1973).

$$\sigma_w/u_* \simeq \phi_m^{-1} \qquad (10)$$

For an absolutely neutral atmosphere, equation (10) implies equality of $\sigma_w$ and $u_*$. However, the near neutral ($0 > z/L > -0.08$) measurements at Kansas indicate a $\sigma_w$ to $u_*$ ratio between 1.2 and 1.3, which is consistent with other near neutral observations (Pasquill, 1974). The situation is further clouded by Wieringa's demurral to a sigma w correction factor for the Kansas data. A $\sigma_w$ to $u_*$ ratio of 1.2 was adopted for the near neutral condition, pending further clarification.

Using equations (4) through (10), a generalized expression is found for the unstable regime by linking vertical turbulence ($\sigma_e$, radians) to meteorological profiles. Equation (11) describes this relationship. For near

$$\sigma_e = \sigma_w/u \simeq k\phi_m^{-1}/(\ln(z/z_0) - \psi) \qquad (11)$$

neutral conditions, equation (12) pertains.

$$\sigma_e = \sigma_w/u \simeq 1.2k/(\ln(z/z_0) - \psi) \qquad (12)$$

No relationships comparable to equations (11) and (12) were found for stable regime data. However, Waldron (1980) used Project Prairie Grass stable regime data to establish an empirical regression equation for the 2-meter bulk Richardson number and sigma e, as in equation (13).

$$\sigma_e = EXP(-0.2977BU + 1.416) \qquad (13)$$

## VI. RESOLVING VARIATIONS IN i

Ideal turbulence measurements would be made by a perfectly responding instrument during a steady state period of unbounded random turbulence. Cloud growth under these ideal conditions would be described as a linear function of the travel distance-turbulence product. In reality, instrument response is imperfect and turbulence varies in space and time. To compensate for departures from ideal conditions, equation (14) introduces a variable correction factor (i). Sigma z can be readily computed using equation (14)

$$\sigma_z = \sigma_e x i \qquad (14)$$

once i is described as a function of travel distance and meteorological profile data.

To depict the functional relationship of i to travel distance and meteorological profile data, a field of i values distributed over a range of stabilities and travel distances was needed. This field of i values was obtained by solving equation (14) for i using Project Prairie Grass data. Sigma z and sigma e values were computed at each trial arc distance using procedures described in III and V above. The correction factor computed in this way compensates for departures from ideal turbulence measurements and deviations from the assumptions implied by equation (1). This procedure is useful so long as the resultant field of i values can be described as a function of the parameters responsible for producing the measured dosages. It was assumed that, with corrections for continuity, source configuration, and roughness, the relationships derived using Project Prairie Grass data would be applicable to other data sets.

Real values of i were calculated for all arc distances on Project Prairie Grass trials for which data were available. A derivation in Appendix A shows that the upper limit of real i values is reached when $CWID \cdot \bar{u}/Q$ equals 0.323. Beyond this limit values of i are imaginary and cannot be used for evaluation.

Analysis of the field of i values indicated a systematic variation with travel distance, but with considerable scatter between trials. Much of this scatter was found to be a function of a non-dimensional stability parameter, the bulk Richardson number. The 2 meter values of BU provided the best fit. Two meter bulk Richardson numbers were computed from profile data using equation (15).

$$BU = 100gz^2(\Delta T/\Delta z + 0.01)/Tu^2 \qquad (15)$$

8

where:

   T = 2 meter temperature (degrees Kelvin)

   u = 2 meter wind speed (meters per second)

   z = height of wind speed measurement, 2 meters

$\Delta T/\Delta z$ = ½ to 4 meter temperature differential (degrees Celsius per meter)

   During analysis of the relationship between i and BU it became apparent that the variation of i with BU assumed an arctangent shape. The arctangent shape allows for large changes in i as BU passes through near neutral stability, with decreasing changes in i as BU approaches the stability extremes. Also, the amplitude of the fitted arctangent curves appear to increase systematically with distance. Equation (16) was developed to describe this expanding topological field of i values over a continuum of stability and distance.

$$i = (-\text{ATAN}(a \cdot BU + b) + c)/d \tag{16}$$

where:

$$a = 10.0 \tag{17}$$

$$b = x^{0.21} - 0.9 \tag{18}$$

$$c = 3980x^{-1.6} + 1.5708 \tag{19}$$

$$d = 1860x^{-1.2} + 0.37 \tag{20}$$

   Variables a through d in equation (16) allow for adjustment of the expanding arctangent field. Equations (17) through (20) define a "best fit" field, subject to several limiting conditions. The first limiting condition is that the coefficient "a" must be constant for all distances. Any variation in "a" with distance would imply a cross correlation, or a functional relationship, between BU and travel distance. The second limiting condition is that i can never become zero or negative. Because an arctangent is limited in range of radian values to ±1.5708, the minimum value of "c" must be 1.5708. The remaining coefficient values were selected by trial and error to provide the best fit to the i values derived from Project Prairie Grass data.

   Figure 1 depicts a segment of the i field given by equation (16) for arc distances of 50 to 800 meters and a BU range of ±2.0. The most noticeable feature on this figure is the dramatic change in i for small changes in the near neutral values of BU, illustrating the marked transition in diffusing power as the atmosphere passes between stable and unstable regimes. Figure 1 also illustrates expansion of the arctangent relationship between BU and i as travel distance increases. Also, Figure 1 shows correction factor values to be small and decreasing with travel distance for the stable regime, and large and increasing with travel distance for the unstable regime. Equations (17) through (20) allow for infinite adjustment of this topological field to fit the array of Project Prairie Grass i values.
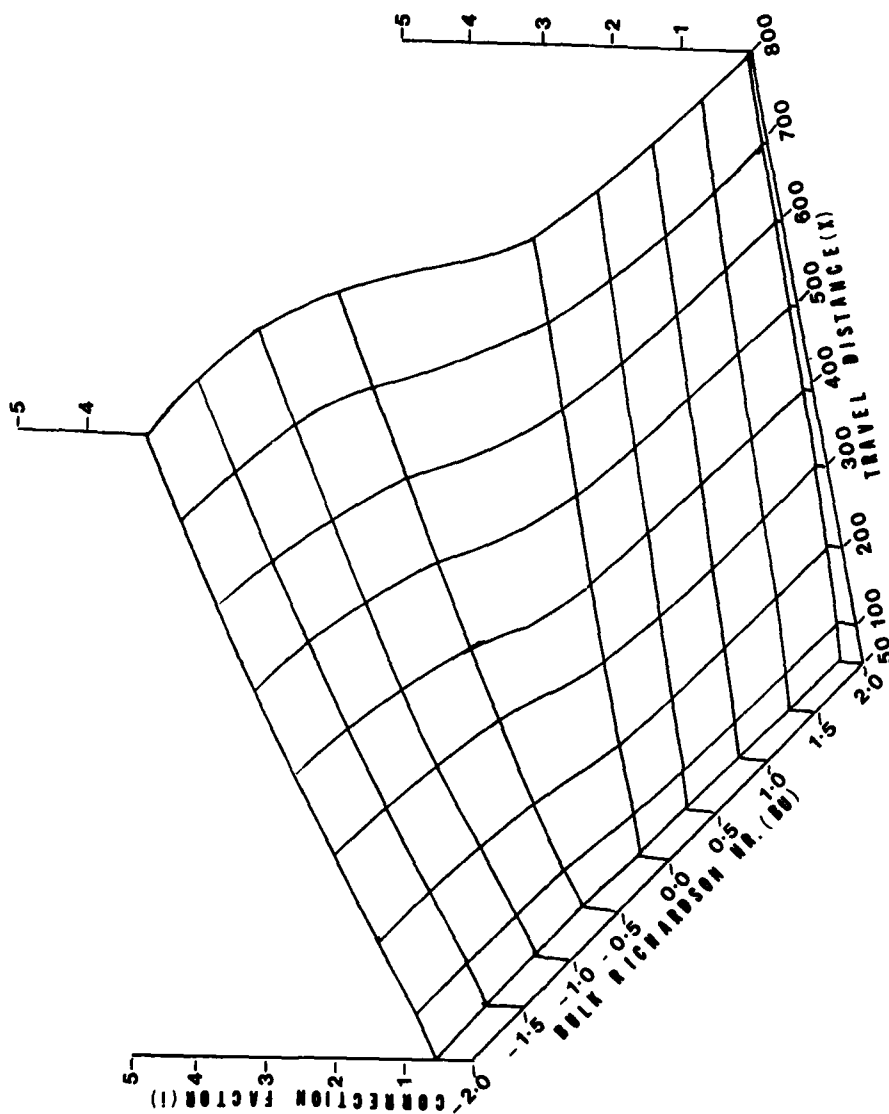
9

FIGURE 1. A Segment of the Expanding Arctangent Field Described by $i = (-ATAN(a \cdot BU + b) + c)/d$

10

## VII. VERIFICATION ON DEPENDENT DATA

As a check on the sigma z computation procedure, a verification was performed with the Project Prairie Grass data. Sigma z values were computed from equation (14) using profile derived sigma e values and correction factors from equation (16). These sigmas were then used in equation (1) for re-computation of crosswind integrated dosages, designated as $CWID_p$. The $CWID_p$ were then compared to the original CWIDs obtained from sampler arc average concentration measurements. Fractional error (FE) defined by equation (21) was used as the basis for comparison of $CWID_p$ to CWID.

$$FE = (CWID_p - CWID)/\tfrac{1}{2}(CWID_p + CWID) \tag{21}$$

Fractional error is the difference between observed and predicted CWIDs, normalized by the average of observed plus predicted CWIDs. Horst (1979) described FE computed in this way as providing a logarithmically unbiased means of presenting the fractional difference between observed and predicted parameter values. The maximum possible fractional difference between two parameters produces a FE of 2.0, and a factor of 2 difference is indicated by a FE of 0.667. Mean fractional error (FE) is defined by equation (22), where N is the number of trials evaluated. Root mean square fractional error

$$\overline{FE} = \sum FE/N \tag{22}$$

(FErms) is defined by equation (23).

$$FErms = (\sum FE^2/N)^{\tfrac{1}{2}} \tag{23}$$

There were 71 Project Prairie Grass trials. No $SO_2$ concentration data were available for trials 48S2, 63, and 64. With five arc distances per trial, 340 cases remained. Concentration data were missing at 800 meters and 400 meters for trials 39 and 40 respectively, and trial 3 was eliminated because most of the cloud missed the samplers, leaving a total of 333 cases for evaluation. Results of the fractional error evaluation for the remaining cases are presented in Table 1. Pasquill stability categories used in Table 1 were determined by Golder's (1972) procedure. Results presented in Table 1 compare favorably with those obtained by Horst (1979) in his dimensionless parameter treatment of the same data. Less than 4 percent of the $CWID_p$ differed from CWID by a factor of 2 or more. The largest fractional errors were generally found at the 800-meter arc and stability categories A and F. Overpredictions of $CWID_p$ during unstable conditions at 800 meters were probably due to cloud centroid rise. Overpredictions of $CWID_p$ during stable conditions may be due to cloud travel along a laminar sublayer, skirting the samplers at 1.5 meters.

11

Table 1. Mean and Root Mean Square Fractional Error for Project Prairie Grass Data at Indicated Distances and Pasquill Stability Categories.

### Mean Fractional Error

| Distance (meters) | Pasquill Stability Category | | | | | | |
|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | ALL |
| 50 | -0.17 | -0.08 | -0.15 | -0.01 | -0.12 | 0.28 | -0.03 |
| 100 | 0.02 | -0.06 | -0.07 | 0.15 | -0.04 | 0.15 | 0.05 |
| 200 | 0.07 | -0.26 | -0.06 | 0.18 | 0.03 | 0.11 | 0.06 |
| 400 | -0.04 | -0.25 | 0.01 | 0.11 | -0.09 | 0.07 | 0.02 |
| 800 | 0.40 | -0.05 | 0.15 | -0.13 | -0.40 | 0.15 | -0.03 |
| ALL | 0.05 | -0.14 | -0.02 | 0.06 | -0.12 | 0.15 | 0.01 |

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

### Root Mean Square Fractional Error

| Distance (meters) | Pasquill Stability Category | | | | | | |
|---|---|---|---|---|---|---|---|
| | A | B | C | D | E | F | ALL |
| 50 | 0.37 | 0.47 | 0.22 | 0.09 | 0.13 | 0.42 | 0.26 |
| 100 | 0.36 | 0.43 | 0.20 | 0.19 | 0.12 | 0.33 | 0.25 |
| 200 | 0.35 | 0.43 | 0.27 | 0.23 | 0.12 | 0.36 | 0.28 |
| 400 | 0.26 | 0.47 | 0.32 | 0.19 | 0.18 | 0.55 | 0.32 |
| 800 | 0.57 | 0.65 | 0.29 | 0.27 | 0.46 | 0.45 | 0.39 |
| ALL | 0.39 | 0.50 | 0.26 | 0.20 | 0.24 | 0.42 | 0.30 |

## VIII. VERIFICATION ON INDEPENDENT DATA

After successful application to Project Prairie Grass data, the analytic method was applied to independent sets of data. The first of these consisted of twenty three surface source trials conducted at the Horizontal Grid on the U. S. Army Dugway Proving Ground, Utah. These trials are known as the BC 412 Trials (Cramer et al., 1965). A major advantage of using BC 412 data for verification is that a large number of vertical concentration profiles were obtained from an extensive vertical array of samplers. Direct vertical concentration measurement obviated source strength considerations, as the sigma z was determined directly from the vertical profiles. Deposition was assumed to be insignificant for the short travel distances, as 1 - 10 μm *Bacillus subtilis* var *niger* particles were used as a tracer. It was also assumed that all samplers had similar efficiencies and were subjected to similar handling and assay techniques. Source generation was accomplished using trailer mounted disseminators pulled along a track perpendicular to the wind direction. It was determined that this dissemination procedure produced a cloud with a source sigma of 1.75 meters and a release height of approximately 1.5 meters above ground level.

Meteorological profile data accompanying the BC 412 Trials were collected on 30 meter towers instrumented for temperature and wind profile measurements. The ½ to 4 meter temperature differentials were determined from trial profile data, and 2 meter bulk Richardson numbers were calculated. The unstable regime BU were processed through equations discussed in Section V to produce estimates of sigma e. Estimates of sigma e for the stable regime trials were obtained using equation (13) modified by a roughness correction (Smith, 1972). The roughness correction is the ratio of DPG Horizontal Grid to Project Prairie Grass $z/z_0$ logarithms. For a DPG Horizontal Grid $z_0$ of 2.3 centimeters, a Project Prairie Grass $z_0$ of 0.8 centimeters, and a height of 2 meters, the roughness correction is given by equation (24). The stable

$$\ln(2/0.023)/\ln(2/0.008) = 1.24 \tag{24}$$

regime sigma e regression equation with the roughness correction are given by equation (25). Sigma z was computed as the square root of the sum of

$$\sigma_e = (EXP(-0.2977BU + 1.416)) \cdot 1.24 \tag{25}$$

the source and turbulent growth variances, as provided in equation (26).

$$\sigma_z = (1.75^2 + (\sigma_e x i)^2)^{\frac{1}{2}} \tag{26}$$

13

Results of verification with BC 412 data are presented in Table 2. Fractional errors were computed for the predicted versus measured sigma z using the procedure applied during Project Prairie Grass CWI$\Gamma_p$ verification. The FErms for the entire data set was 0.31, and 6 percent of the predicted sigma z differed from measured values by more than a factor of 2.

Table 2. Mean and Root Mean Square Fractional Errors for Computed versus Observed Sigma Z, DPG Horizontal Grid BC 412 Data.

| Distance | Unstable Regime | | | Stable Regime | | |
|----------|------|-------|---|------|-------|----|
| (meters) | FE | FErms | N | FE | FErms | N |
| 30 | -0.20 | 0.25 | 8 | -0.09 | 0.32 | 15 |
| 91 | -0.05 | 0.15 | 8 | -0.19 | 0.32 | 15 |
| 274 | 0.36 | 0.55 | 5 | -0.13 | 0.28 | 15 |
| 427 | 0.22 | 0.34 | 3 | 0.05 | 0.26 | 15 |
| 793 | ---- | ---- | 0 | 0.25 | 0.35 | 14 |
| ALL | 0.02 | 0.33 | 24 | -0.03 | 0.31 | 74 |

A second evaluation on an independent data set was performed using data collected at the National Reactor Testing Station (NRTS) in southeast Idaho (Islitzer and Dumbauld, 1963). Diffusion measurements were made with sampling arcs operated 100, 200, 400, 800, 1600, and 3200 meters downwind from the source. The source was a uranine dye in small-particulate form released at a height of 1 meter for 60 minutes per trial. Crosswind integrated concentration data were reported for each arc, normalized to a source strength of 1 gram per second and a transport wind of 5 meters per second. The cloud centroid and measurement height were both assumed to be at the surface ($z = h = 0.0$).

The NRTS test site was described as flat with widely distributed sagebrush. Roughness was estimated at 1.5 centimeters. Meteorological profile measurements of temperature gradients and wind velocities were made near the tracer release point. Tabulated 4-and 8-meter Ri were converted to z/L and reduced to 2-meter values. For the unstable regime, this allowed the use of the same stability parameters as were applied to Project Prairie Grass data. No stable regime evaluations were performed because insufficient data were available for calculation of comparable stability parameters.

Evaluation of continuity considerations for the NRTS trials was difficult because tracer deposition was suspected (Islitzer and Dumbauld, 1963). Equation (27), attributed by Islitzer and Dumbauld to Chamberlain, was used to compensate for loss of source strength with downwind travel. Islitzer and Dumbauld found a $V_0$ of 5.2 centimeters per second and a $n_z$ of -0.24 to be

$$Q_c = Q \cdot EXP(-4V_0 x/((\sqrt{2}\pi)n_z \tilde{u}\sigma_z))$$ (27)

where:

$Q_c$ = adjusted source strength (grams)

$V_0$ = deposition velocity (meters per second)

$n_z$ = dispersion parameter (dimensionless)

average values for these parameters. Sufficient data were not available to use equation (3) for $\tilde{u}$ calculation. Therefore, the Islitzer and Dumbauld tabulated wind speed data were used for $\tilde{u}$ in deposition calculations.

Table 3 provides the results of fractional error evaluation for observed versus predicted crosswind integrated concentrations with unstable regime NRTS data.

Table 3. Mean and Root Mean Square Fractional Errors for Computed versus Observed Crosswind Integrated Concentrations, NRTS Data.

| Distance (meters) | Unstable Regime | | |
|---|---|---|---|
| | FE | FErms | N |
| 100 | -0.03 | 0.45 | 14 |
| 200 | -0.26 | 0.48 | 23 |
| 400 | -0.16 | 0.48 | 13 |
| 800 | -0.05 | 0.45 | 22 |
| 1600 | 0.13 | 0.45 | 14 |
| 3200 | 0.48 | 0.68 | 14 |
| ALL | -0.01 | 0.50 | 100 |

The FErms of 0.50 is higher than that for the Project Prairie Grass or the BC 412 data, but is well within a factor of two. Twenty percent of the cases were in error by a factor of two or more. The largest errors were found at the 3200-meter distance, where systematic overprediction occurred. These overpredictions may be due to insufficient deposition calculated with equation (26), insufficient cloud growth prediction using $\tilde{i}$, or some combination of the two.

## IX. CONCLUSIONS

An analytic method for calculating sigma z from meteorological profile data has been developed and demonstrated. With appropriate adjustments for source sigmas, deposition, and surface roughness, the method was successfully used on independent data sets, consistently providing estimates of sigma z or crosswind integrated dosage well within a factor of 2 accuracy.

The sigma z calculation method is based on the use of a correction factor adjustment to rectilinear cloud growth. For Project Prairie Grass data, this correction factor was a function of stability and travel distance, described over a continuum of stability and travel distance by an expanding arctangent equation. Because cloud growth at other locations observes the same functional relationship to stability and travel distance, the method has universal applications. One must note, however, that the correction factor field developed in this report is based on a surface source and sampling at 1.5 meters above ground level. Changes in source or sampling height may require adjustments to the correction factor field pattern. Sufficient data to define such adjustments are not available.

The sigma z calculation method requires an estimate of vertical turbulence, and procedures for estimating vertical turbulence from meteorological profile data are demonstrated. The turbulence estimation procedure for the unstable regime is valid through a z/L of -1.0. Application of this procedure in extremely unstable conditions where z/L exceeds -1.0 is uncertain because adequate relationships between stability parameters have not been developed for these conditions. Likewise, use of the regression equation which describes sigma e as a function of BU becomes uncertain beyond a BU of +4.0. Also, relationships between stability parameters are not well established for the stable regime, requiring a known 2-meter BU before stable regime sigma e values can be estimated.

Although the sigma z calculation method is applicable over the full range of stability and travel distance, present procedures for estimating vertical turbulence are subject to revision. Fortunately, coefficients of the arctangent equation are infinitely adjustable. As new turbulence computation procedures are developed, coefficients of the arctangent equation will allow for adjustments to accomodate revisions while retaining the functional form.

16

REFERENCES

Barad, M. L., (Ed.) 1958a: Project Prairie Grass, a field program in
diffusion. Vol. I, GRP No. 59, AFCRC-TR-58-235 (I), Air Force
Cambridge Research Laboratories, Bedford, Mass. 280 pp.

-----, (Ed.) 1958b: Project Prairie Grass, a field program in diffusion.
Vol. II, GRP No. 59, AFCRC-TR-58-235 (II), Air Force Cambridge
Research Laboratories, Bedford, Mass. 209 pp.

-----, and D. A. Haugen, 1959: A preliminary analysis of Sutton's
hypothesis for diffusion from a continuous point source. J.
Meteor., 16, 12-20.

Businger, J. A., 1973: Turbulent transfer in the atmospheric surface
layer. Workshop in Micrometeorology (D. A. Haugen, Ed.)., 67-100.

Cramer, H. E., G. M. DeSanto, K. R. Dumbauld, B. P. Greene, P. Morgenstern,
and R. N. Swanson, 1965: CB field test prediction trials. GCA Tech.
Rpt. No. 65-9-G, U. S. Army Dugway Proving Ground, Dugway, Utah,
184 pp. (AD-646 334)

-----, J. R. Bjorklund, R. K. Dumbauld, J. E. Faulkner, F. A. Record, P. N.
Swanson, and A. G. Tingle, 1972: Development of dosage models and
concepts. GCA Tech. Rpt. No. 70-15-G, U. S. Army Deseret Test Center,
Fort Douglas, Utah. (AD-893 341L)

Dyer, A. J., and B. B. Hicks, 1970: Flux-gradient relationships in the
constant flux layer. Quart. J. R. Meteor. Soc., 96, 715-721.

Dyer, A. J., 1974: A review of flux-profile relationships. Boundary-Layer
Meteorol., 7, 363-372.

Elliott, W. P., 1961: The vertical diffusion of gas from a continuous
source. Int. J. Air and Water Poll., 4, 33-46.

Golder, D., 1972: Relations among stability parameters in the surface
layer. Boundary-Layer Meteorol., 3, 47-58.

Haugen, D. A., (Ed.) 1959: Project Prairie Grass, a field program in
diffusion. Vol. III, GRP No. 59, AFCRC-TR-58-235 (III), Air Force
Cambridge Research Laboratories, Bedford, Mass. 673 pp.

Horst, T. W., 1979: Lagrangian similarity modeling of vertical diffusion
from a ground-level source. J. Appl. Meteor., 18, 733-740.

Islitzer, N. F., and R. K. Dumbauld, 1963: Atmospheric diffusion-deposition
studies over flat terrain. Int. J. Air and Water Poll., 7, 999-1022.

Izumi, Y., (Ed.) 1971: Kansas 1968 field program data report. Environm. Res. Pap. No. 379, AFCRL-72-0041, Air Force Cambridge Research Laboratories, Bedford, Mass. 73 pp.

Pasquill, F., 1974: Atmospheric Diffusion. John Wiley and Sons, New York, 429 pp.

-----, 1978: Atmospheric dispersion parameters in plume modeling. EPA Rpt. No. 600/4-78-021. U. S. Environmental Protection Agency, Research Triangel Park, North Carolina. (NTIS No. PB-283 655)

Paulson, C. A., 1970: The mathematical representation of wind speed and temperature profiles in the unstable atmospheric surface layer. J. Appl. Meteor., 9, 857-861.

Smith, F. B., 1972: A scheme for estimating the vertical dispersion of a plume from a source near ground level. Proceedings of the third meeting of the expert panel on air pollution modeling, Oct 2-3, 1972, Paris, France. NATO Committee on the Challenges of Modern Society, NATO/CCMS Pub. No. 14. Brussels, Belgium.

Sutton, O. G., 1953: Micrometeorology. McGraw-Hill, New York, 333 pp.

Waldron, A. W., 1980: personal communication.

Willis, G. E., and J. W. Deardorff, 1976: A laboratory model of diffusion into the convective planetary boundary layer. Quart. J. R. Meteor. Soc., 102, 427-445.

Wieringa, J., 1980: A revaluation of the Kansas mast influence on measurements of stress and cup anemometer overspeeding. Boundary-Layer Meteorol., 18, 411-430.

## DERIVATION OF A $\dfrac{\widetilde{CWID\,U}}{Q}$ LIMIT FOR THE CWID EQUATION

The basic CWID equation including reflection terms is

$$CWID = \frac{Q}{\sqrt{2\pi}\,\sigma_z \widetilde{U}} \left[ e\left(\frac{-(z-h)^2}{2\sigma_z^2}\right) + e\left(\frac{-(z+h)^2}{2\sigma_z^2}\right) \right] \tag{A1}$$

Substituting $\sigma_w ti$ for $\sigma_z$ and rearrangement of terms yields:

$$\frac{\sqrt{2\pi}\ CWID\ \sigma_w t \widetilde{U}}{Q} = \frac{1}{i}\left[ e\left(\frac{-(z-h)^2}{2\sigma_z^2}\right) + e\left(\frac{-(z+h)^2}{2\sigma_z^2}\right) \right] \tag{A2}$$

with

$$(z+h)^2 = (z-h)^2 + 4zh \tag{A3}$$

and defining

$$A = \frac{1}{2\sigma_w^2 t^2}\ , \tag{A4}$$

$$B = \frac{\sqrt{2\pi}\ CWID\ \sigma_w t \widetilde{U}}{Q}\ , \tag{A5}$$

then,

$$B = \frac{1}{i}\left[ e\left(\frac{-A(z-h)^2}{i^2}\right) + e\left(\frac{-A((z-h)^2+4zh)}{i^2}\right) \right] \tag{A6}$$

Considering $i$ a general differentiable variable, and simplifying equation (A6) by defining

$$C = A(z-h)^2, \tag{A7}$$

$$D = A((z-h)^2 + 4zh), \tag{A8}$$

$$F(i) = \frac{1}{i}\left[ e^{(-C/i^2)} + e^{(-D/i^2)} \right] - B = 0 \tag{A9}$$

Taking the derivative,

$$\frac{df(i)}{di} = \frac{d}{di}(\frac{1}{i})[e^{(-C/i^2)} + e^{(-D/i^2)}] + \frac{1}{i}\frac{d}{di}[e^{(-C/i^2)} + e^{(-D/i^2)}] \tag{A10}$$

At the maximum value of a function, the derivative is zero,

$$\frac{df(i)}{di} = 0 \tag{A11}$$

Simplifying equation A10 and gathering terms,

$$i^2\frac{df(i)}{di} = (\frac{2C}{i^2} - 1) e^{(-C/i^2)} + (\frac{2D}{i^2} - 1) e^{(-D/i^2)} = 0 \tag{A12}$$

For a sampling height ($z$) of 1.5 meters and cloud centroid height ($h$) at 0.5 meters,

$$(z-h)^2 = 1.0, \tag{A13}$$

$$4zh = 3.0 \tag{A14}$$

Then,

$$i^2\frac{df(i)}{di} = (\frac{2A}{i^2} - 1) e^{(-A/i^2)} + (\frac{8A}{i^2} - 1) e^{(-4A/i^2)} = 0 \tag{A15}$$

As long as $NA/i^2$ is small,

$$e^{NA/i^2} \approx 1 + NA/i^2 \tag{A16}$$

Then,

$$(2A/i^2 - 1)(1 - A/i^2) + (8A/i^2 - 1)(1 - 4A/i^2) = 0 \tag{A17}$$

Simplifying,

$$-34A^2 + 15Ai^2 - 2i^4 = 0 \tag{A18}$$

Solving equation (A18) quadratically,

$$i^2 = \frac{15A}{4} \pm \frac{6.855}{4}\sqrt{-A^2} \tag{A19}$$

A-2

It can be shown by De Moivre's theorum that

$$i^4 = (14.0625 + 2.937)A^2 \qquad\qquad\qquad (A20)$$

Hence:

$$i^2 = 4.123A \qquad\qquad\qquad (A21)$$

Substituting equation (A21) into equation (A6), along with equations (A13) and (A14) yields

$$B = \frac{0.5731}{\sqrt{A}} \qquad\qquad\qquad (A22)$$

From equation (A4) one obtains

$$\frac{1}{\sqrt{A}} = \sqrt{2}\sigma_w t \qquad\qquad\qquad (A23)$$

Then, employing equations (A22), (A23), and (A5),

$$B = \frac{0.5731}{\sqrt{A}} = 0.5731\sqrt{2}\sigma_w t = \frac{\sqrt{2\pi}\ CWID\ \sigma_w t \overrightarrow{U}}{Q} \qquad\qquad\qquad (A24)$$

Or,

$$\frac{CWID\ \overrightarrow{U}}{Q} = 0.3233, \qquad\qquad\qquad (A25)$$

which is very close to the exact solution achieved iteratively (0.3235).

Any $CWID\ \overrightarrow{U}/Q$ value >0.3233 violates the limit [equation (A11)] on equation (A10) and therefore provides no real solution to the $CWID$ equation. The limit presented in equation (A25) is unique for the given values of $z$ and $h$, but the procedure can be used to solve for other $z$ and $h$ values. For a condition where the derivative is nonzero, it is apparent that there are two possible solutions for equation (A19). This situation can be resolved by choosing the smaller real value for $i$.

DATE
ILME